USING TRIANGULATIONS IN COMPUTER SIMULATIONS OF GRANULAR MEDIA

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ABSTRACT

The distinct element method used to simulate behavior of granular media intrinsically needs a lot of computation. We developed a data structure based on a weighted Delaunay triangulation to efficiently detect collisions between a large number of discs. This triangulation is locally maintained in the course of time. It defines an implicit neighborhood which allows faster computations. Several numerical experiments have been carried out. Some interesting phenomena have been observed, for instance segregation and convective processes induced by vibrations.

KEYWORDS

Granular media, distinct element method (DEM), rigid-particle model, left to right triangulation, weighted Delaunay triangulation, computational geometry, computer simulation, segregation, convective streams.

INTRODUCTION

A granular medium is a large set of elements, called grains, that interact. We can find many examples of such media in different fields : nature (sand, soil, Rings of Saturn, ...), alimentation (salt, pepper, Bircher Muesli, ...), civil engineering (railroad beds, dams, roads, ...), chemistry, pharmacy (mixtures). Many of their properties are still not well known and simulation has become an interesting tool for studying them, since computers are now powerful enough.

To simulate these media, we use a Newtonian discrete model. That means we model each grain separately and use Newtonian physics to calculate their movements. To manage the collisions there exist two approaches : the rigid-particle model, and the soft-particle model like the famous model invented by Cundall et Strack (1979). In the first instance, the grains cannot overlap, what means we need to know time and point of impact. In the other, we observe the overlaps at regular short time intervals, and these overlaps are used to calculate the forces and the new velocities after integration. We will present here a rigid-particle model, but the triangulation technique can easily be adapted for soft-particle models.

TRIANGULATIONS

In this paper we chose to represent grains with discs, but Müller and Liebling (1994) have developed a similar triangulation technique with polygons. Figure 1 shows a weighted Delaunay triangulation constructed on the centers of gravity of discs. It has an important property : when two discs are in contact they are always linked by an edge. To identify the neighbors of a disc, we just have to examine its edges.

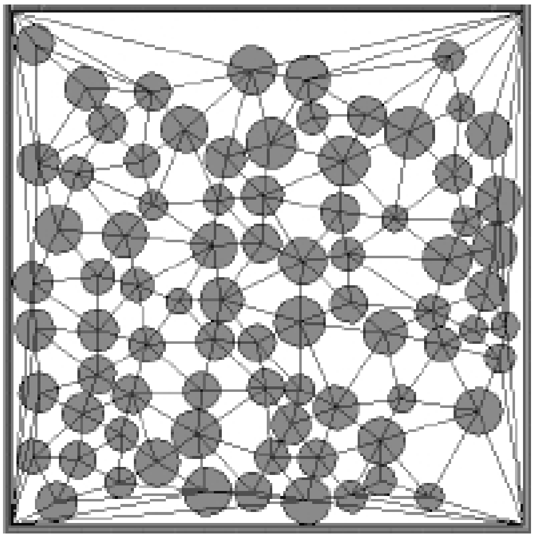


Fig. 1. Weighted Delaunay triangulation.

The construction of this triangulation is made in two steps. First, we construct any triangulation, for instance scanning the centers of gravity sorted from left to right and connecting them to those on their left side without crossing an existing edge, as presented in fig.2. The four corners of the surrounding box are considered to be the discs of radius zero.

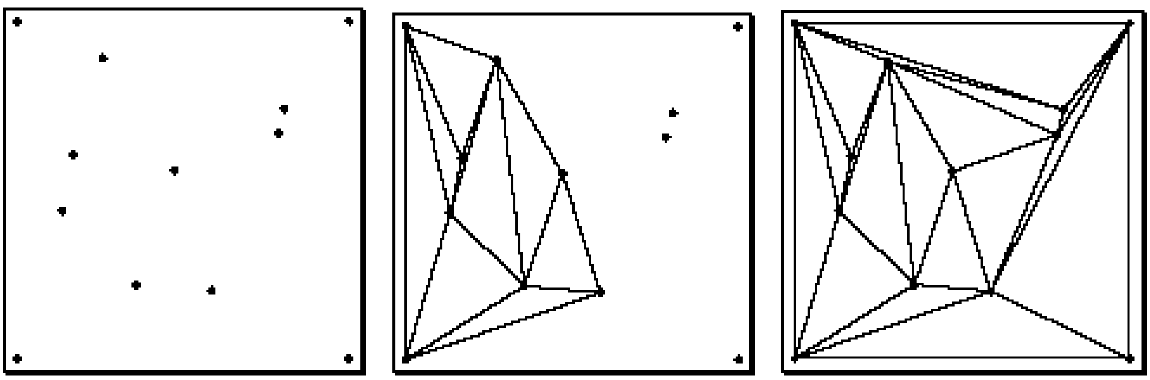


Fig. 2. Triangulation built from left to right

To simplify, we will first explain how to transform any triangulation into a Delaunay triangulation, and afterwards present the minor modification necessary to obtain the weighted one. Starting from the initial triangulation, we modify it by using a sequence of local operations called *flips*. A flip of an edge represents an exchange of diagonals of a convex quadrilateral. After a flip we still have a triangulation.

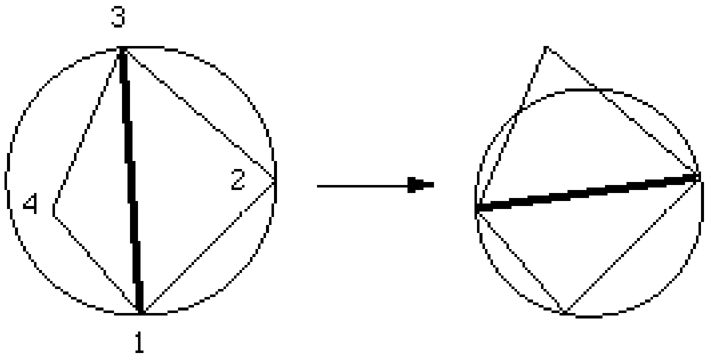


Fig. 3. Flip

In a Delaunay triangulation, we have the following property : take both adjacent triangles to a given edge. The circle that circumscribes any of these triangles must not contain the fourth vertex. To quickly detect whether the fourth vertex is inside the circle, we proceed by projecting the circle onto a paraboloid. In the affirmative the projected point is below the plane containing the projection. According to the convexity property, this works for any paraboloid.

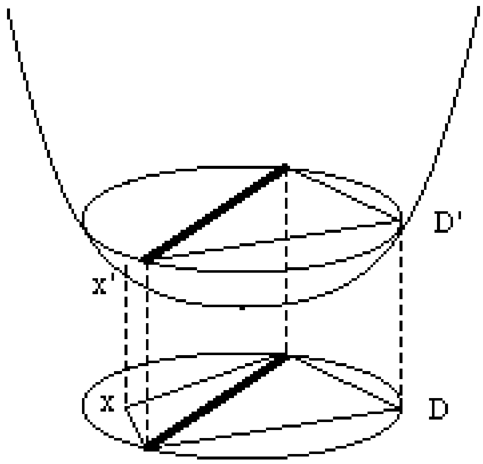


Fig. 4. Projection onto a paraboloid

Thus computing a simple determinant and determining its sign is enough to decide whether the fourth point is above the plane. For a weighted Delaunay triangulation, we replace points by discs and we only have to subtract the square of the disc radius from the z-coordinate of the projected points (see formula (1)).

$$\Delta e = \begin{vmatrix} x_1 & y_1 & (x_1^2 - y_1^2) - r_1^2 & 1 \\ x_2 & y_2 & (x_2^2 - y_2^2) - r_2^2 & 1 \\ x_3 & y_3 & (x_3^2 - y_3^2) - r_3^2 & 1 \\ x_4 & y_4 & (x_4^2 - y_4^2) - r_4^2 & 1 \end{vmatrix} \qquad (1)$$

It is very important to have only additions and multiplications to verify the legality of edges, because we have to repeat this test many times. If we used square roots or trigonometric functions, it would seriously slow down the simulation.

The main idea of the algorithm is to flip all illegal edges until all edges are legal. First we scan all edges of the triangulation and stack all illegal edges. Then we flip all edges in the stack, but after each flip we must verify if the four edges of the induced quadrilateral are still legal and put the illegal ones into the stack. The algorithm stops when the stack is empty.

Now the discs can move according to Newtonian laws. In the process sometimes the triangulation has to be modified. Rather than redo it entirely, we only repair it wherever needed. Since we know motion equations of the disc, we can calculate where and when an edge will become illegal by replacing the fixed coordinates in the determinant by coordinates that depend on time. An edge becoming illegal is the first type of event we will encounter.

We also have to calculate collision times, either between two discs or between a disc and a wall. All these events sorted by occurence time are placed in a schedule. This schedule is not definitive, it has to be updated every time an event occurs. For example, when two discs collide, we have to recalculate the flip time of every edge that touches these discs and collision times of both discs. To calculate the new velocities after a shock, we can use the formulas given by Wang & Mason (1992) or by Hogue & Newland (1993). To this end, we have to know the impact point and, for both discs, the velocities at the impact time, the moment of inertia and two physical parameters : the friction and restitution coefficients. In these models, overlaps are prohibited. That is why we must proceed from one event time to the next. It is not adequate to examine the configuration at regular time intervals, because mentioned formulas concern only two discs at the same time.

NUMERICAL SIMULATIONS

The simulated experiments consist of shaking a 1x1 m box that contains a thousand discs with diameters between 10 and 20 mm. Friction and restitution coefficients, μ and e, are between 0.4 and 0.6. Density is 2.6. We will shake the box in different ways and we hope to observe some convective motions that occur in real life. The oscillations are of type $A \cdot \sin(2\pi f t)$. Each experiment lasts one minute in reality and about sixty hours on a SiliconGraphics workstation with a 100 MHz processor.

In the trial we present here, there are vertical oscillations with a shearing motion of the walls. That means the vertical walls move up and down and in opposite directions. Coloring the discs in two colors facilitates observations. Moreover we added three big discs to follow their behavior. It is well known that when we shake a box containing a granular medium the big grains move to the surface. But it is not clear *how* they do it. As wee can see in fig. 5, the big grains may also move to the bottom, carried by the induced streams shown at the last row of table 1. But when they are on the surface, their large size prevents them from going down again, except when they are located close to the walls. In the last snapshot, we can observe the two big clear-colored discs stuck at the confluence of two streams.

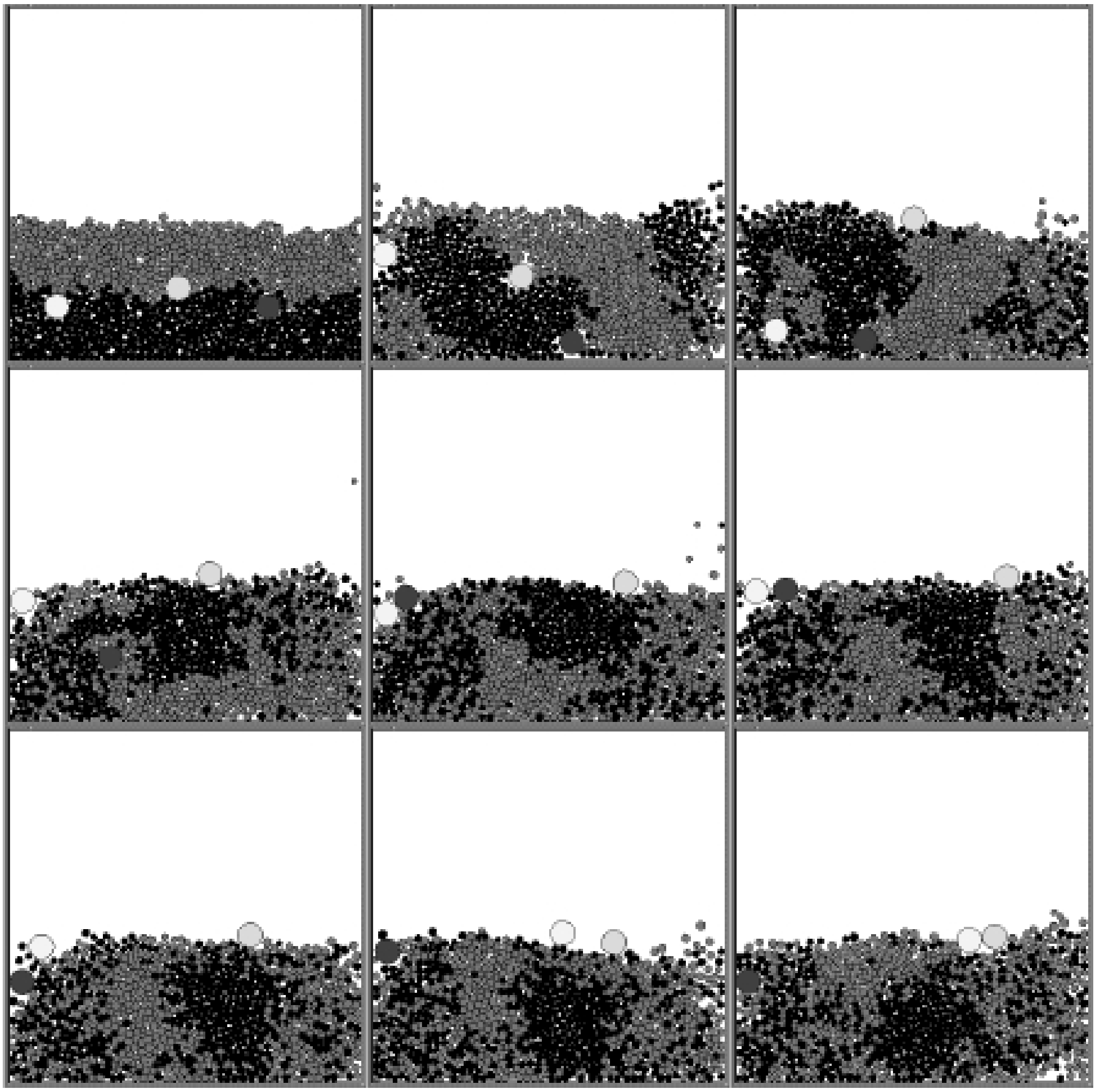
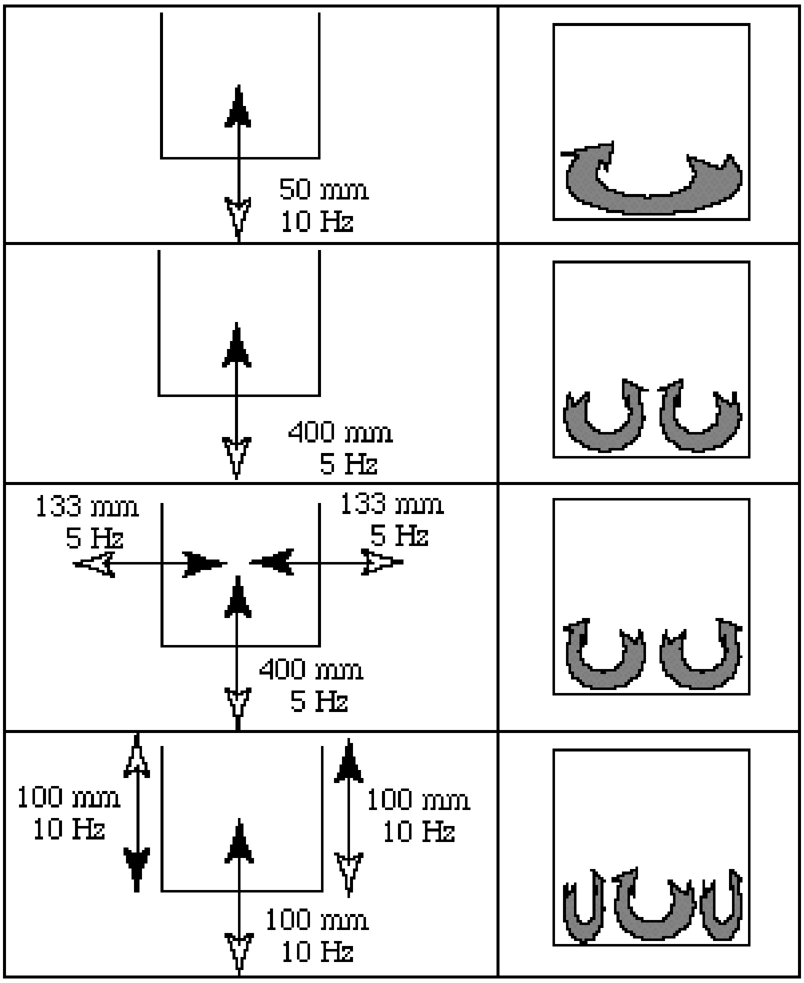


Fig. 5. Snapshots of a simulation

In fig. 5 we can observe three convective streams. It is easier to observe such on the animations realized by Müller (1994). Generally, only one or two streams will appear. Table 1. summarizes four observed types of behavior. The third case is rare and seemingly requires particular initial conditions. During our series of experiments it appeared only when the discs were initially placed on a grid. In the other cases, grains were poured as shown in the first image of fig. 5.

Table 1. Summary of observed behaviors.



The mechanism of convection is not well understood. In particular, it is difficult to predict the number and the direction of the streams. Hopefully further computer simulations will allow to analyze this phenomenon more accurately.

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