

Calcul de la somme de n termes (S_n) d'une progression arithmétique

$$\begin{array}{r} S_n = t_1 + t_2 + t_3 + \dots + t_{n-2} + t_{n-1} + t_n \\ + S_n = t_n + t_{n-1} + t_{n-2} + \dots + t_3 + t_2 + t_1 \\ \hline 2S_n = (t_1 + t_n) + \underbrace{(t_2 + t_{n-1})}_{t_1 + r + t_n - r} + \underbrace{(t_3 + t_{n-2})}_{t_1 + 2r + t_n - 2r} + \dots + \underbrace{(t_{n-2} + t_3)}_{t_n - 2r + t_1 + 2r} + \underbrace{(t_{n-1} + t_2)}_{t_n - r + t_1 + r} + \underbrace{(t_n + t_1)}_{t_1 + t_n} \\ \underbrace{\phantom{(t_2 + t_{n-1})}}_{t_1 + t_n} \underbrace{\phantom{(t_3 + t_{n-2})}}_{t_1 + t_n} \dots \underbrace{\phantom{(t_{n-2} + t_3)}}_{t_1 + t_n} \underbrace{\phantom{(t_{n-1} + t_2)}}_{t_1 + t_n} \underbrace{}_{t_1 + t_n} \end{array}$$

RESULTAT : $2S_n = n \cdot (t_1 + t_n) \Rightarrow S_n = \frac{n}{2} (t_1 + t_n) = \frac{n}{2} (2t_1 + (n - 1) \cdot r)$